CS103
Winter 2026

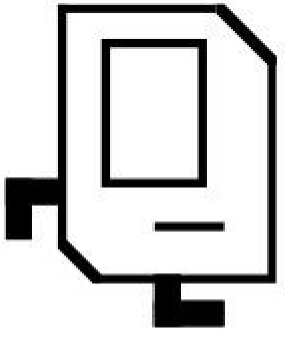
Lecture 08:

Set Theory Revisited

Set Theory Revisited

1. **Section Leading!**
2. Quick Recap
3. How Not to Prove Things About Sets
4. Definitions (and the Assume-Prove Table for Sets)
5. Proofs on Subsets
6. Unions and Intersections
7. Announcements
8. Set Equality
9. Set-Builder Notation
10. Power Set Membership
11. Action Items
12. What’s Next?
13. Appendix: Additional Set Proof Examples

CS198 Section Leading



cs198@cs.stanford.edu

Who should section lead?

For this round of applications, we are looking for applicants have completed the equivalent of CS106B!

If you’re currently taking CS106B, there will be a later application cycle.

We are looking for section leaders from all backgrounds who can relate to students and clearly explain concepts.

What do section leaders do?

- Teach a weekly 50 minute section
- Help students in the LaIR
- Grade CS106 assignments
- Hold IGs with students
- Grade midterms and finals
- Get paid $18.50/hour (more with seniority)
- Have fun!

Time and requirements

You'll need to:

- Section lead for **two quarters!**
- Take CS198 for 3-4 units (1st quarter only)
- Attend staff meetings (Monday, 4:30-5:30PM)
- Attend Monday workshops (7:30-9pm) for first 4 weeks of first quarter
- Attend Wednesday workshops (based on availability) for first 4 weeks of first quarter
- Fulfill all teaching, LaIR, and grading responsibilities

Why section lead?

- “Learn to teach; teach to learn”
- Work directly with students
- Participate in fun events
- Join an amazing group of people
- Leave your mark on campus
- Fun Fact: more than half of the 103 teaching staff are former SLs!

Participate in fun events

- LaIR Formal
- Special D
- BAWK
- Lecturer Hangouts
- New SL Picnic
- Swag
- And more!

Apply Now

Application is open now!

Deadline if you have already completed CS106B:
Monday, January 26th at 11:59PM

Deadline if you are currently taking CS106B:

Monday, February 9th at 11:59PM

Online application: cs198.stanford.edu

Contact us: cs198@cs.stanford.edu

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The Assume-Prove Table

	If you ***assume*** this is true...	To ***prove*** that this is true...
$\forall x.\ A$	Initially, ***do nothing***. Once you find a z through other means, you can state it has property A.	Have the reader pick an arbitrary x. We then prove A is true for that choice of x.
$\exists x.\ A$	Introduce a variable x into your proof that has property A.	Find an x where A is true. Then prove A is true for that specific choice of x.
$A \to B$	Initially, ***do nothing***. Once you know A is true, you can conclude B is also true.	Assume A is true, then prove B is true.
$A \land B$	Assume A. Also assume B.	Prove A. Also prove B.
$A \lor B$	Consider two cases. Case 1: A is true. Case 2: B is true.	Either prove $\neg A \to B$ or prove $\neg B \to A$. *(Why does this work?)*
$A \leftrightarrow B$	Assume $A \to B$ and $B \to A$.	Prove $A \to B$ and $B \to A$.
$\neg A$	Simplify the negation, then consult this table on the result.	Simplify the negation, then consult this table on the result.

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Claim: If A, B, and C are sets where $A \in B$ and $B \in C$, then $A \in C$.

Proof (?): Assume A, B, and C are sets where $A \in B$ and $B \in C$. We need to show that $A \in C$.

Since $A \in B$, we know that A is contained in B. Since $B \in C$, we know that B is contained in C. Therefore, because A is contained in B and B is contained in C, we know that A is contained is C. This means that $A \in C$, as required. ■

Claim: If A, B, and C are sets where $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

Proof (?): Assume A, B, and C are sets where $A \subseteq B$ and $B \subseteq C$. We need to show that $A \subseteq C$.

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Which of these claims are true (if any)?

Answer at
cs103.stanford.edu/pollev

Lie: If A, B, and C are sets where $A \in B$ and $B \in C$, then $A \in C$.

Bad Proof: Assume A, B, and C are sets where $A \in B$ and $B \in C$. We need to show that $A \in C$.

Since $A \in B$, we know that A is contained in B. Since $B \in C$, we know that B is contained in C. Therefore, because A is contained in B and B is contained in C, we know that A is contained is C. This means that $A \in C$, as required. ■

Theorem: If A, B, and C are sets where $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

Bad Proof: Assume A, B, and C are sets where $A \subseteq B$ and $B \subseteq C$. We need to show that $A \subseteq C$.

Since $A \subseteq B$, we know that A is contained in B. Since $B \subseteq C$, we know that B is contained in C. Therefore, because A is contained in B and B is contained in C, we know that A is contained is C. This means that $A \subseteq C$, as required. ■

This can't be a good proof; the same basic argument proves a false claim!

Claim: Let A, B, and C be sets. If $A \subseteq B$ and $A \subseteq C$, then we have $A \subseteq B \cap C$.

Proof (?): Assume $A \subseteq B$ and $A \subseteq C$. We need to show $A \subseteq B \cap C$.

Since $A \subseteq B$, all elements of A are in B. Since $A \subseteq C$, all elements of A are also in C. Therefore, all elements of A are in both B and C. Therefore, we see that $A \subseteq B \cap C$. ■

Claim: Let A, B, and C be sets. If $A \subsetneq B$ and $A \subsetneq C$, then we have $A \subsetneq B \cap C$.

Proof (?): Assume $A \subsetneq B$ and $A \subsetneq C$. We need to show $A \subsetneq B \cap C$.

Since $A \subsetneq B$, all elements of A are in B and there are other elements of B. Since $A \subsetneq C$, all elements of A are also in C and there are other elements of C. Therefore, all elements of A are in both B and C, and there are some other elements in B and C. Therefore, we see that $A \subsetneq B \cap C$. ■

(Reminder: $S \subsetneq T$ means $S \subseteq T$ and $S \neq T$.)

Which of these claims are true (if any)?

Answer at
cs103.stanford.edu/pollev

Theorem: Let A, B, and C be sets. If $A \subseteq B$ and $A \subseteq C$, then we have $A \subseteq B \cap C$.

Bad Proof: Assume $A \subseteq B$ and $A \subseteq C$. We need to show $A \subseteq B \cap C$.

Since $A \subseteq B$, all elements of A are in B. Since $A \subseteq C$, all elements of A are also in C. Therefore, all elements of A are in both B and C. Therefore, we see that $A \subseteq B \cap C$. ■

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Bad Proof: Assume $A \subsetneq B$ and $A \subsetneq C$. We need to show $A \subsetneq B \cap C$.

Since $A \subsetneq B$, all elements of A are in B and there are other elements of B. Since $A \subsetneq C$, all elements of A are also in C and there are other elements of C. Therefore, all elements of A are in both B and C, and there are some other elements in B and C. Therefore, we see that $A \subsetneq B \cap C$. ■

(Reminder: $S \subsetneq T$ means $S \subseteq T$ and $S \neq T$.)

What Went Wrong?

- The style of arguments you've just seen are ***not*** how to prove results on sets.
- Here are two particular things that went wrong:
  - The reliance on high-level terms like "contained" is not mathematically precise.
  - A discussion of "all elements" of a set is not how to reason about collections of objects.
- ***Question:*** How do we write rigorous proofs about sets?

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The Importance of Definitions

- As you've seen in recent lectures, formal definitions underpin mathematical proofs.
- The major issue from the previous proofs is that we haven't defined what our terms mean.
    - How do we define what $A \in B$ means?
    - How do we define what $A \subseteq B$ means?
    - How do we define what $A \cap B$ means?
- Think back to our proof writing triad: we currently have intuitions for these concepts, but not formal definitions.

	Is defined as...	If you ***assume*** this is true...	To ***prove*** that this is true...
$S \subseteq T$			
$S = T$			
$x \in S \cap T$			
$x \in S \cup T$			
$X \in \wp(S)$			
$x \in \{ y \mid P(y) \}$			

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Theorem: If A, B, and C are sets where $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

Defining Subsets

- Formally speaking, if S and T are sets, we say that $S \subseteq T$ when the following holds:

$$\forall x \in S.\ x \in T$$

- Now, suppose you're working with a proof where you encounter $S \subseteq T$. Think back to the proof table.
  - To ***assume*** that $S \subseteq T$, what should you do?
  - To ***prove*** that $S \subseteq T$, what should you do?

Answer at

cs103.stanford.edu/pollev

	Is defined as…	If you ***assume*** this is true…	To ***prove*** that this is true…
$S \subseteq T$	$\forall x \in S.\ x \in T$	Initially, ***do nothing***. Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
$S = T$			
$x \in S \cap T$			
$x \in S \cup T$			
$X \in \wp(S)$			
$x \in \{ y \mid P(y) \}$			

A Correct Proof on Sets

Theorem: If A, B, and C are sets where $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

Proof: Let A, B, and C be sets where $A \subseteq B$ and $B \subseteq C$. We need to prove that $A \subseteq C$. To do so, pick some $x \in A$; we need to show that $x \in C$.

Since $x \in A$ and $A \subseteq B$, we know $x \in B$. Similarly, since $x \in B$ and $B \subseteq C$, we see that $x \in C$, as required. ■

Let's compare this to our previous approach.

Theorem: If A, B, and C are sets where $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

Proof: Let A, B, and C be sets where $A \subseteq B$ and $B \subseteq C$. We need to prove that $A \subseteq C$. To do so, pick some $x \in A$; we need to show that $x \in C$.

Since $x \in A$ and $A \subseteq B$, we know $x \in B$. Similarly, since $x \in B$ and $B \subseteq C$, we see that $x \in C$, as required. ■

Bad Proof: Assume A, B, and C are sets where $A \subseteq B$ and $B \subseteq C$. We need to show that $A \subseteq C$.

Since $A \subseteq B$, we know that A is contained in B. Since $B \subseteq C$, we know that B is contained in C. Therefore, because A is contained in B and B is contained in C, we know that A is contained is C. This means that $A \subseteq C$, as required. ■

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Theorem: Let A, B, and C be sets. If $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

Unions and Intersections

- The statement $x \in S \cap T$ is defined as

$$\boldsymbol{x \in S \quad \wedge \quad x \in T.}$$

- The statement $x \in S \cup T$ is defined as

$$\boldsymbol{x \in S \quad \vee \quad x \in T.}$$

- These are operational definitions: they show how unions and intersections interact with the ∈ relation rather than saying what the union or intersection of two sets "are."

	Is defined as…	If you ***assume*** this is true…	To ***prove*** that this is true…
$S \subseteq T$	$\forall x \in S.\ x \in T$	Initially, ***do nothing***. Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
$S = T$			
$x \in S \cap T$	$x \in S \ \land \ x \in T$	Assume $x \in S$. Then assume $x \in T$.	Prove $x \in S$. Also prove $x \in T$.
$x \in S \cup T$	$x \in S \ \lor \ x \in T$	Consider two cases: Case 1: $x \in S$. Case 2: $x \in T$.	Either prove $x \in S$ or prove $x \in T$.
$X \in \wp(S)$			
$x \in \{ y \mid P(y) \}$			

Theorem: Let A, B, and C be sets. If $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

Proof: Assume $A \subseteq B$ and $A \subseteq C$. We need to prove that $A \subseteq B \cap C$. So pick some $x \in A$; we need to show that $x \in B \cap C$.

Since $x \in A$ and $A \subseteq B$, we know $x \in B$. Since $x \in A$ and $A \subseteq C$, we know $x \in C$. Therefore, we see that $x \in B$ and $x \in C$, so $x \in B \cap C$, as required. ■

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Midterm Exam Logistics

- Our first midterm exam is next ***Tuesday, February 3rd***, from ***7–10 PM***. Locations vary.
- You're responsible for Lectures 00 – 05 and topics covered in PS1 – PS2, and the assume-prove table material from lectures 06 – 07. Later topics (functions forward) and problem sets (PS3 onward) won't be tested here, other than topics related to the assume-prove table. Exam problems may build on the written or coding components from the problem sets.
- The exam is closed-book, closed-computer, and limited-note. You can bring a double-sided, 8.5" × 11" sheet of notes with you to the exam, decorated however you'd like.

Midterm Exam Logistics

- Students with alternate exam arrangements: you should have heard from us with details. If you haven't, contact Anisha and Sean ASAP.
- Later today, we'll post an ***Exam Logistics*** page on the course website with full details and logistics.
- It will also include advice from former CS103 students about how to do well on the exams.
- Check it out – there will be tons of goodies on

Exam Day Logistics

- We’ll have proctors in the room.
- We have assigned seating; see course website. These will be posted Wednesday.
- Check out your seat in advance, and screenshot it!
- No phones, calculators, or other digital devices during the exam.
- No belongings near your seats (leave them in your dorms or at the front of the classroom).
- Must have Stanford student ID to turn in exam.

Extra Practice Problems

- Later today, you'll find Extra Practice Problems 1 on the website: a collection of practice midterms and an assortment of other questions.
- ***Our Recommendation:***
  - Work through one or two practice exams under ***realistic conditions*** (block off three hours, have your notes sheet, use pencil and paper).
  - Review the solutions only when you're done. ***Don't peek!*** You can't do that on the actual exam.
  - Ping the course staff to ask questions, whether that's "please review this proof I wrote for one of the exam questions" or "why doesn't the solution do X, which seems easier than Y, which is what it did?"
  - ***Internalize the feedback***. What areas do you need more practice with? Study up on those topics. What transferrable skills did you learn in the course of solving the problems? If you aren't sure, ask!
  - Repeat!
- Realistically, we don't expect you to do all of the practice exams. We've provided those just so you can get a sense of what's out there.

Review Session

- One or more of our amazing CAs, will be holding a review session this ***Thursday, January 29th***. Time and location will be posted on Ed!
- Come prepared to discuss any questions you may have.
- You'll get more out of this session if you have done some preliminary study first.

Final Thoughts

- We want everyone to do well on the exam!
- Let us know if you have any questions as we approach the exam – whether logistical in nature or content-related.

Also!

- One guide and one checklist unlocking today.
- Attendance opt-out form is due Friday by 11:59 PM.

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Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

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$S \subseteq T$	$\forall x \in S.\ x \in T$	Initially, ***do nothing***. Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
$S = T$	$S \subseteq T \land T \subseteq S$	Assume $S \subseteq T$ and $T \subseteq S$.	Prove $S \subseteq T$. Also prove $T \subseteq S$.
$x \in S \cap T$	$x \in S \land x \in T$	Assume $x \in S$. Then assume $x \in T$.	Prove $x \in S$. Also prove $x \in T$.
$x \in S \cup T$	$x \in S \lor x \in T$	Consider two cases: Case 1: $x \in S$. Case 2: $x \in T$.	Either prove $x \in S$ or prove $x \in T$.
$X \in \wp(S)$			
$x \in \{ y \mid P(y) \}$			

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: See appendix!

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Set-Builder Notation

- Let S be the set defined here:

$$S = \{ n \mid n \in \mathbb{N} \text{ and } n \geq 137 \}$$

- Now imagine you have some quantity x. Based on this…
  - … if you ***assume*** that $x \in S$, what does that tell you about x?
  - … if you need to ***prove*** that $x \in S$, what do you need to prove?

Answer at

cs103.stanford.edu/pollev

Set-Builder Notation

- Like unions and intersections, we have an operational definition for set-builder notation. It's the following:

Let $S = \{ y \mid P(y) \}$.
Then $x \in S$ when $P(x)$ is true.

- So, for example:
  - $x \in \{ n \mid n \in \mathbb{N}$ and n is even $\}$ means $x \in \mathbb{N}$ and x is even.
  - $x \in \{ n \mid \exists k \in \mathbb{N}.\ n = 2k + 1 \}$ means that there is a $k \in \mathbb{N}$ where $x = 2k + 1$. (Equivalently, x is an odd natural number)
- ***Key Point:*** The placeholder variable disappears in all these examples. After all, *it's just a placeholder.*

	Is defined as…	If you ***assume*** this is true…	To ***prove*** that this is true…
$S \subseteq T$	$\forall x \in S.\ x \in T$	Initially, ***do nothing***. Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
$S = T$	$S \subseteq T \land T \subseteq S$	Assume $S \subseteq T$ and $T \subseteq S$.	Prove $S \subseteq T$. Also prove $T \subseteq S$.
$x \in S \cap T$	$x \in S \land x \in T$	Assume $x \in S$. Then assume $x \in T$.	Prove $x \in S$. Also prove $x \in T$.
$x \in S \cup T$	$x \in S \lor x \in T$	Consider two cases: Case 1: $x \in S$. Case 2: $x \in T$.	Either prove $x \in S$ or prove $x \in T$.
$X \in \wp(S)$			
$x \in \{ y \mid P(y) \}$	$P(x)$	Assume $P(x)$.	Prove $P(x)$.

Some Useful Notation

- If n is a natural number, we define the set **[*n*]** as follows:

$$[n] = \{ k \mid k \in \mathbb{N} \land k < n \}$$

- So, for example:
  - $[3] = \{0, 1, 2\}$
  - $[0] = \emptyset$
  - $[5] = \{0, 1, 2, 3, 4\}$

Theorem: If $m, n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$

Theorem: If $m, n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$

What We're Assuming	*What We Need to Prove*
$m \in \mathbb{N}$	~~$[m] \subseteq [n]$~~
$n \in \mathbb{N}$	~~$\forall x \in [m].\ x \in [n]$~~
$m < n$	$x \in \mathbb{N}$
$[z] = \{ k \mid k \in \mathbb{N} \land k < z \}$	$x < n$
$x \in [m]$	
$x \in \mathbb{N}$	
$x < m$	

Theorem: If $m, n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$.

Proof: Assume m and n are natural numbers where $m < n$. We need to show that $[m] \subseteq [n]$. To do so, pick some $x \in [m]$. We'll prove that $x \in [n]$.

Since $x \in [m]$, we know that $x \in \mathbb{N}$ and $x < m$. Then, because $x < m$ and $m < n$, we know that $x < n$. Collectively this means that $x \in \mathbb{N}$ and $x < n$, so $x \in [n]$, as required. ■

Notice that *there is no set-builder notation in this proof*. We were able to avoid it by using the rules for what $x \in \{ y \mid P(y) \}$ say to do.

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$x \in S \cap T$	$x \in S \land x \in T$	Assume $x \in S$. Then assume $x \in T$.	Prove $x \in S$. Also prove $x \in T$.
$x \in S \cup T$	$x \in S \lor x \in T$	Consider two cases: Case 1: $x \in S$. Case 2: $x \in T$.	Either prove $x \in S$ or prove $x \in T$.
$X \in \wp(S)$	$X \subseteq S$.	Assume $X \subseteq S$.	Prove $X \subseteq S$.
$x \in \{ y \mid P(y) \}$	$P(x)$	Assume $P(x)$.	Prove $P(x)$.

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Your Action Items

- ***Read “Guide to Proofs on Discrete Structures.”***
  - There’s additional guidance and practice on the assume/prove dichotomy and how it manifests in problem-solving.
- ***Read “Discrete Structures Proofwriting Checklist.”***
  - Keep the items here in mind when writing proofs. We’ll use this when grading your problem set.
- ***Read “Guide to Proofs on Sets.”***
  - There’s some good worked examples in there to supplement today’s lecture, several of which will be relevant for the problem set.
- ***Start Problem Set 3.***
  - Start early and make slow and steady progress.
- ***(Optional) Look over Extra Practice Problems 1***
  - For extra pre-exam review. These will be posted later today.

Set Theory Revisited

Next Time

- ***Graph Theory***
  - A ubiquitous, powerful abstraction with applications throughout computer science.
- ***Vertex Covers***
  - Making sure tourists don’t get lost.
- ***Independent Sets***
  - Helping the recovery of the California Condor.

Set Theory Revisited

1. Section Leading!
2. Quick Recap
3. How Not to Prove Things About Sets
4. Definitions (and the Assume-Prove Table for Sets)
5. Proofs on Subsets
6. Unions and Intersections
7. Announcements
8. Set Equality
9. Set-Builder Notation
10. Power Set Membership
11. Action Items
12. What’s Next?
13. **Appendix: Additional Set Proof Examples**

Appendix:

More Sample Set Proofs

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$. We consider each case separately.

Case 1: $x \in A$. Since $A \subseteq C$ and $x \in A$, we see that $x \in C$, and therefore that $x \in C \cup D$.

Case 2: $x \in B$. Then because $B \subseteq D$ and $x \in B$ we have $x \in D$, so $x \in C \cup D$.

In either case, we see that $x \in C \cup D$, as required. ■

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$. Since $x \in B$, we know that $x \in A \cup B$, as required.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$. So pick an $x \in A$; we need to show that $x \in B$.

Since $x \in A$, we know that $x \in A \cup B$. And since $x \in A \cup B$ and $A \cup B = B$, we see that $x \in B$, as required. ■

Theorem: Let A and B be sets. Then if $\wp(A) = \wp(B)$, then $A = B$.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$. Since the roles of A and B are symmetric, we'll just prove $A \subseteq B$.

Pick some $x \in A$; we need to show that $x \in B$. Since $x \in A$, we know that $\{x\} \subseteq A$. This means that $\{x\} \in \wp(A)$, and since $\wp(A) \subseteq \wp(B)$ we know $\{x\} \in \wp(B)$. Thus we see that $\{x\} \subseteq B$, which in turn means that $x \in B$, as required. ■